AH– 1543 CV-19-S

(052) M.Sc. (Previous) Mathematics
Examination 2019-20
Compulsory/Optional

Paper-VI

DIFFERENTIAL GEOMETRY OF MANIFOLDS

Time: Three Hours] [Maximum Marks: 100
[Minimum Passing Marks:-036

Note: Answer all questions. All question carry equal marks

1. a. Show that tangent bundle is a Vector bundle.

   b. Show that the cartesion product of differentiable manifolds are again differentiable manifold.

2. State and prove generalized Gauss and Mainrdicodazzi equation.

3. a. Prove that D(1) = 2D(1)

   b. Prove that $X_x = [T_x U]^{-1}.T_x U.X_x$

4. Show that the circle S'C¢ is a Lie group under complex multiplication and the map

$$Z = e^{i\theta} \rightarrow \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & In-2 \end{pmatrix}$$

   Is a Lie group homomorphism into so(n)

5. Let (M,g) be a Riemann manifold with Sectional Curvature $k \geq k_0 > 0$ Then show that for any geodesic C in M, the distance between two conjugate points along C is $\leq \frac{\pi}{\sqrt{k_0}}$

6. Let $g: L \rightarrow \theta$ *be a surjective submersion which is proper, show that* $g^{-1}(k)$ *is compact in L for each compact KcQ and Let Q be connected, Then show that (L, P, Q) is fibre bundle*

7. a. Explain almost complex manifolds with example.

   b. Prove that a Riemannian manifold (m,g) is geodesically complete if and only if D(p) = T_pm

8. a. Write a short Note about differentiable manifold, and give an example of it.

   b. Define following

      i. Induced Bundle

      ii. Nijenhuis Tensor

      iii. Hyper surfaces.

9. a. Show that the range of the Zero section of a Vector Bundle $E \rightarrow M$ is a sub manifold of E.

   b. State an prove Local immersion theorem.

10. a. *Let* $y_1: I_1 \rightarrow M$ and $y_2: I_2 \rightarrow M$ *are geodesics and* $y_1(a) = y_2(a)$, $y_1^1 \ (a) = y_2^1(a)$ *for some* $a \in I_1 \cap I_2$ *then prove that* $y_1 = y_2$ *on* $I_1 \cap I_2$

    b. Prove that any element of compact and connected Lie Group Contained in some maximal torus.